## RESEARCH PAPERS

# A Synchrotron X-ray Study of the Electron Density in $\mathbf{Y}_{\mathbf{2}} \mathbf{B a C u O}_{5}$ 

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#### Abstract

The deformation density ( $\Delta \rho$ ) for $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$, barium diyttrium cuprate, determined by single-crystal X-ray diffraction with synchrotron radiation, is affected to only a limited degree by the bonding interactions involving the O anions. Electron density is strongly depleted along the cation-cation contacts within the mirror plane in the structure and is transferred to regions between mirror planes that do not lie along short cation-cation vectors. The structural geometry for the $\mathrm{CuO}_{5}$ moiety, with the Cu atom in the +2 state, closely resembles that of the $\mathrm{Cu}_{2} \mathrm{O}_{5}$ group in $\mathrm{YBa}_{2} \mathrm{Cu}_{3} \mathrm{O}_{7-\delta}$, for which the +3 state involvement for Cu has been suggested. Space group Pnma, orthorhombic, $M_{r}=458.68, a=12.1793$ (7), $b=$ $5.6591(5), \quad c=7.1323$ (4) $\AA, \quad V=491.6 \AA^{3}, \quad Z=$ $4, D_{x}=6.197 \mathrm{Mg} \mathrm{m}^{-3}, \lambda=0.9 \AA, \mu_{0.9}=28.893 \mathrm{~mm}^{-1}$, $F(000)=812, T=293 \mathrm{~K}, R=0.020, w R=0.020, S=$ 3.09 (5) for 2225 unique reflections.


## 1. Introduction

To help explain the high-temperature superconductivity of $\mathrm{YBa}_{2} \mathrm{Cu}_{3} \mathrm{O}_{7-\delta}$ (Bednorz \& Mueller, 1986; Wu, Ashburn, Torng, Hor, Meng, Gao, Huang, Wang \& Chu, 1987), Cava, Batlogg, van Dover, Murphy, Sunshine, Siegrist, Remeika, Rietman, Zahurak \& Espinosa, (1987) studied the related semi-conducting greenish oxide $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$, which often occurs as an impurity (Weidinger, Budnick, Chamberland, Golnik, Niedermayer, Recknagel, Rossmanith \& Yang, 1988). The magnetic ordering of $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ at 15 K may be related to the superconducting phase transition in $\mathrm{YBa}_{2} \mathrm{Cu}_{3} \mathrm{O}_{7-\delta}$.

Some hypotheses relate the high-temperature superconductivity of the 123 compound to the oxidation number of the Cu 2 atom. Charge balance for the oxidation states of +2 for $\mathrm{Ba},+3$ for Y and -2 for O requires that Cu in $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ be in a +2 state, whereas that for Cu 2 in $\mathrm{YBa}_{2} \mathrm{Cu}_{3} \mathrm{O}_{7-\delta}$ is frequently cited as exceeding +2 when $\delta$ falls below 0.5 . Because $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ is unaffected by non-stoichiometry or disorder, its struc-
tural geometry and $\Delta \rho$ topography provide reference standards with which those for Cu 2 in $\mathrm{YBa}_{2} \mathrm{Cu}_{3} \mathrm{O}_{7-8}$ can be compared.

An approximate X-ray powder diffraction structure analysis for $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ by Michel \& Raveau (1982) was confirmed by single-crystal X-ray diffraction (Hazen, Finger, Angel, Prewitt, Ross, Mao, Hadidiacos, Hor, Meng \& Chu, 1987; Watkins, Fronczek, Wheelock, Goodrich, Hamilton \& Johnson, 1988; Sato \& Nakada, 1989). Distorted monocapped trigonal $\mathrm{YO}_{7}$ prisms that share one triangular face form $\mathrm{Y}_{2} \mathrm{O}_{11}$ blocks. The $\mathrm{Ba}^{2+}$ and $\mathrm{Cu}^{2+}$ cations lie in cavities in the three-dimensional edge-sharing network of $\mathrm{Y}_{2} \mathrm{O}_{11}$ units. Each Ba atom is coordinated to 11 O atoms. The Cu atoms are located within distorted $\mathrm{CuO}_{5}$ square pyramids. The low atomsite symmetries in these metal oxides allow their bonding and second nearest-neighbour interactions to be compared. This is more difficult for compounds where the bonding and second nearest-neighbour interactions have symmetry elements in common.

The electron density in $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ single crystals was measured by Buttner \& Maslen (1993) using Mo $K \alpha$ radiation. The $\Delta \rho$ topography near the Cu atom resembled that near Cu 2 in $\mathrm{YBa}_{2} \mathrm{Cu}_{3} \mathrm{O}_{7-\delta}$, which has similar coordination, but noise in that study made it difficult to draw firm conclusions on the longer range interactions that may be important in superconductivity. This noise can be reduced using synchrotron radiation.

## 2. Experimental

A crystal suitable for accurate diffraction imaging with sharp edges and well formed (101̄), (101), (010) and ( $\overline{2} 01$ ) faces was measured with a Philips 505 scanning electron microscope (SEM) at 1100 magnification to be 22.0 (3), 47.7 (3), 258.9 (3) and 17.0 (3) $\mu \mathrm{m}$, respectively, from an internal origin. A bright red $\mathrm{He}-\mathrm{Ne}$ laser beam ( $\lambda=632.8 \mathrm{~nm}$ ) grating calibration determined the $0.3 \mu \mathrm{~m}$ accuracy.

Intensity data were measured accurately at the Tsukuba Photon Factory by J. R. Hester, E. N.

Maslen and V. A. Streltsov, with the crystal mounted perpendicular to the ( 010 ) faces. The $\lambda=0.9 \AA$ radiation from the beamline 14 A vertical wiggler was polarized strongly in the vertical direction (Satow \& Iitaka, 1989). A double-crystal monochromator with a double-focusing mirror is coincident with the equatorial plane of the four-circle diffractometer in a horizontal plane.

The diffraction profiles for five $h 00$, four $0 k 0$ and three $00 l$ reflections were essentially single peaks, except at the centres of the 400,020 and 002 reflections, for which the NaI scintillation counter's linear range of 20000 counts $\mathrm{s}^{-1}$ was exceeded so markedly that their profiles had double peaks. An attenuator, introduced automatically during data collection when the peak count rate exceeded a preset value, reduced strong intensities by the attenuation factor of 34.0 to within the range for which analytical dead time corrections were reliable.

Cell dimensions were determined from measured angles for six reflections ( $79.0<2 \theta<105.3^{\circ}$ ). The crystal was re-oriented during data collection when the intensities of standard reflections dropped suddenly by more than $10 \%$, presumably because seismic activity disturbed the alignment.
Absorption corrections were evaluated by the analytical method (Alcock, 1974) with an attenuation coefficient calculated from atomic $\mu / \rho$ values for $\lambda=$ $0.9 \AA$ kindly updated by Creagh (1992). The crystal's elongated, mildly non-centrosymmetric shape was a critical test of the analytical absorption routine's precision. Optical path lengths calculated for some Friedel pairs differed initially, although equality was expected. Close scrutiny of the algorithm for forming Howell's polyhedra after analysing asymptotic forms for the optical path lengths using Mathematica (Wolfram, 1991) allowed the problems to be rectified. The precision of the Xtal3.2 system ABSORB subroutine (Hall, Flack \& Stewart, 1992) was improved to meet the needs of these calculations.

Secondary extinction corrections to $|F|^{2}$ based on Zachariasen's (1967) formalism as applied by Larson (1970) were first assessed by relating intensity differences for symmetry-related reflections to their diffraction path lengths (Spadaccini \& du Boulay, 1992). The isotropic Zachariasen extinction coefficient $r^{*}$ indicated was slightly negative and not significant. Extinction corrections of the $F$ values for the unique reflections, optimized concurrently with the structural parameters by minimizing the disagreement between observed and calculated structure factors by least-squares, were also insignificant. These extinction corrections have the disadvantage of depending on the structural model used to evaluate $F_{c}$.

All calculations utilized the Xtal3.2 system installed on a DECstation 5000/120 computer. The Lorentz and polarization corrections applied assume the doublecrystal monochromator to be perfect. The incident beam polarization factor for a $2 \theta$ monochromator angle of

Table 1. Experimental details

| Crystal data |  |
| :---: | :---: |
| Chemical formula | $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ |
| Chemical formula weight | 458.68 |
| Cell setuing | Orthorhombic |
| Space group | Pnma |
| $a(\mathrm{~A})$ | 12.1793 (7) |
| $b$ ( ${ }^{\text {a }}$ ) | 5.6591 (5) |
| $c\left({ }^{\text {a }}\right.$ ) | 7.1323 (4) |
| $V\left(\dot{A}^{3}\right)$ | 491.59 (6) |
| 2 | 4 |
| $D_{x}\left(\mathrm{Mg} \mathrm{m}^{-3}\right)$ | 6.197 |
| Radiation type | Synchrotron |
| Wavelength ( $\AA$ ) | 0.9000 (1) |
| No. of reflections for cell parameters | 6 |
| $\theta$ range ( ${ }^{\circ}$ ) | 39.510-52.668 |
| $\mu\left(\mathrm{mm}^{-1}\right)$ | 28.893 |
| Temperature (K) | 293 |
| Crystal form | Needle-like |
| Crystal size (mm) | 0.0518 (3) $\times 0.0095$ (3) $\times 0.0034(3)$ |
| Crystal colour | Green |
| Data collection |  |
| Diffractometer | Tsukuba |
| Data collection methou | $\omega / 2 \theta$ |
| Scan speed ( ${ }^{\circ} \mathrm{min}^{-1}$ ) | 16 |
| Peak scan width; $a+b \tan \theta\left({ }^{\circ}\right.$ ) | 0.40; 0.00 |
| $(\sin \theta / \lambda)_{\text {max }}$ | 1.01 |
| Absorption correction | Analytical |
| $T_{\text {min }}$ | 0.55 |
| $\tau_{\text {max }}$ | 0.83 |
| No. of measured reflections | 16154 |
| No. of independent reflections | 2415 |
| No. of observed reflections | 2225 |
| Criterion for observed reflections | $F>\sigma(F)$ |
| $R_{\text {int }}$ | 0.032 |
| $\theta_{\text {max }}\left({ }^{\circ}\right.$ ) | 64.95 |
| Range of h, k.l | $-24 \rightarrow h \rightarrow 24$ |
|  | $-11 \rightarrow k \rightarrow 11$ |
|  | $-14 \rightarrow 1 \rightarrow 14$ |
| No. of standard reflections | 6 |
| Frequency of standard reflections | Every 100 reflections |
| Intensity decay (\%) | 10 |
| Refinement |  |
| Refinement on | $F$ |
| $R$ | 0.020 |
| $w R$ | 0.020 |
| $s$ | 3.092 |
| No. of reflections used in refinement | 2233 |
| No. of parameters used | 49 |
| H -atom treatment | H-atom parameters not refined |
| Weighting scheme | $w^{\prime}=1 / \sigma^{2}(F)$ |
| $(\Delta / \sigma)_{\text {max }}$ | 0.00001 |
| $\Delta \rho_{\text {max }}\left(\mathrm{c} \AA^{-\AA^{-3}}\right.$ ) | 1.504 |
| $\Delta \rho_{\text {min }}\left(\mathrm{e} \AA^{-3}\right.$ ) | -5.234 |
| Extinction method | None |
| Source of atomic scattering factors | International Tables for X-ray Crustallography (1974, Vol IV, Tables 2.2B and 2.3.1) |

$16.57^{\circ}$ was 0.9508 . Atomic form factors were from International Tables for X-ray Crystallography (1974, Vol. IV). The structure-factor calculations included the $\Delta f^{\prime}, \Delta f^{\prime \prime}$ values $-1.346,0.776$ for $\mathrm{Y},-0.063,3.450$ for $\mathrm{Ba}, 0.123,1.943$ for Cu and $0.018,0.010$ for O at $0.9 \AA$, kindly evaluated by Creagh (1992).

Full-matrix least-squares refinement minimized the residual $\Sigma w_{i}\left(\left|F_{o}\right|-\left|F_{c}\right|\right)^{2}$ with $w_{i}=1 / \sigma^{2}(F)$. The accuracy of the weak reflections, with $F_{o}<10 \sigma$ for only

Table 2. Positional and anisotropic vibration parameters $\left(\dot{A}^{2} \times 10^{5}\right)$ for $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ $U_{\text {eq }}(1 / 3) \sum_{i} \sum_{j} U_{i j} a_{i}^{*} a_{j}^{*} \mathbf{a}_{i} \cdot \mathbf{a}_{j}$.

|  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\boldsymbol{x}$ | $y$ | $z$ | $U_{\text {eq }}$ | $U_{11}$ | $U_{22}$ | $U_{33}$ | $U_{12}$ | $U_{13}$ | $U_{23}$ |
| Y 1 | $0.57388(2)$ | 0.75 | $1.10416(3)$ | $371(7)$ | $351(6)$ | $408(7)$ | $352(6)$ | 0 | $-7(5)$ | 0 |
| Y 2 | $0.78823(2)$ | 0.75 | $0.38365(3)$ | $366(7)$ | $339(6)$ | $393(7)$ | $365(6)$ | 0 | $-3(5)$ | 0 |
| Ba | $0.90483(1)$ | 0.75 | $-0.06980(2)$ | $684(5)$ | $599(4)$ | $638(5)$ | $816(5)$ | 0 | $-88(4)$ | 0 |
| Cu | $0.65941(2)$ | 0.75 | $0.71268(4)$ | $426(10)$ | $438(9)$ | $397(9)$ | $442(9)$ | 0 | $112(7)$ | 0 |
| O1 | $0.5675(1)$ | $0.5072(2)$ | $0.8345(2)$ | $599(42)$ | $523(36)$ | $510(42)$ | $765(39)$ | $-62(30)$ | $66(29)$ | $69(31)$ |
| O2 | $0.7724(1)$ | $0.5044(2)$ | $0.6438(2)$ | $724(44)$ | $783(40)$ | $642(44)$ | $748(41)$ | $251(32)$ | $231(31)$ | $230(32)$ |
| O3 | $0.6004(1)$ | 0.75 | $0.4199(2)$ | $669(60)$ | $529(51)$ | $1012(66)$ | $465(50)$ | 0 | $11(41)$ | 0 |

$5 \%$ of the extensive data, is reflected in low refinement indices, listed in Table 1.*

The refined atomic positions listed in Table 2 are fully consistent with those by Michel \& Raveau (1982), Watkins et al. (1988), Sato \& Nakada (1989), Hunter, Town, Davis, Russel \& Taylor (1989), Pei, Paulikas, Veal \& Jorgensen (1990), Salinas-Sanchez, Garcia-Munoz, Rodriguez-Carvajal, Saez-Puche \& Martinez (1992) and Buttner \& Maslen (1993). The vibration amplitudes in Table 2 do not show the anomalously low values for Y1 and Y2 reported by Watkins et al. (1988), being smaller than but closer to those in the X-ray tube study by Buttner \& Maslen (1993). The vibration amplitudes resemble those for $\mathrm{Ba}, \mathrm{Y}$ and Cu 2 in the high- $T_{c}$ compound in so far as Ba is relatively labile, Y is tightly bound and Cu shows intermediate behaviour.

## 3. Structural geometry

The cation coordinates chosen define a near linear $\mathrm{Y} 1 \cdots \mathrm{Cu} \cdots \mathrm{Y} 2 \cdots$ Ba sequence, coplanar with O 3 in the mirror plane at $y=0.75$. All O atoms are near Cu , with O 1 and O 2 displaced below that mirror plane. Table 3 lists important interaction distances. The fivecoordinated Cu atom has two $\mathrm{Cu}-\mathrm{O} 1,1.974$ (1) $\AA$, and two $\mathrm{Cu}-\mathrm{O} 2,2.016$ (1) $\AA$, bonds. The $\mathrm{Cu}-\mathrm{O}$, 2.208 (2) $\AA$, bond lies on the $y=0.75$ mirror plane. The $\mathrm{CuO}_{5}$ geometry resembles that around the Cu 2 atom in the 123 superconducting phase (Buttner, Maslen \& Spadaccini, 1992), as shown in Fig. 1. The elongated pyramid in $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ is less distorted, with the Cu atom displaced by only $0.2268 \AA$ from the oxygen base. The $\mathrm{Cu}-\mathrm{O} 1$ and $\mathrm{Cu}-\mathrm{O} 2$ bonds are tilted from the ideal octahedral orientation by 6.6 and $6.5^{\circ}$, respectively, which are smaller than the $7.7^{\circ}$ for $\mathrm{Cu} 2-\mathrm{O} 2$ and $\mathrm{Cu} 2-\mathrm{O} 3$ in $\mathrm{YBa}_{2} \mathrm{Cu}_{3} \mathrm{O}_{7-\delta}$ (Buttner et al., 1992).

An isolated $\mathrm{Cu}^{2+}$ atom in an octahedral ligand field is unstable against small perturbations, because its $d^{9}$ state is degenerate, as predicted by the Jahn-Teller (Jahn \& Teller, 1937) theorem. Thus, the CuO structure contains elongated O -atom octahedra with Cu at the centre (Langford \& Louer, 1991). The common $D_{4 h}$

[^0]Table 3. Selected interaction-vector lengths and $\mathrm{O}-\mathrm{M}-\mathrm{O}^{\prime}$ angles for $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$
$M$ is $\mathrm{Ba}, \mathrm{Y} 1, \mathrm{Y} 2$ or Cu , with O and $\mathrm{O}^{\prime}$ symmetry-related by reflection across the mirror plane at $y=0.75$.


Symmetry codes: (i) $x, y, 1+z$; (ii) $1-x, \frac{1}{2}+y, 2-z$; (iii) $1-x, 1-y, 2-z$; (iv) $x, y, z$; (v) $x, \frac{3}{2}-y, z$; (vi) $\frac{3}{2}-x, 1-y, \frac{1}{2}+z$; (vii) $\frac{3}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$; (viii) $\frac{3}{2}-x, 1-y, z-\frac{1}{2} ;$ (ix) $\frac{3}{2}-x, \frac{1}{2}+y, z-\frac{1}{2} ; \quad$ (x) $\frac{1}{2}+x, \frac{3}{2}-y, \frac{1}{2}-z$; (xi) $x, y, z-1$; (xii) $x, \frac{3}{2}-y, z-1$; (xiii) $\frac{1}{2}+x, y, \frac{1}{2}-z \quad{ }^{*} M-M$ contacts are within the same $y$ layer.
distortion to the $\mathrm{Cu}^{2+}$ atoms' symmetry has one axis pair of the octahedron elongated. The other two pairs are contracted. The structural geometry of the Cu atom in $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ is roughly of this type. The pyramid base formed by four coplanar O 1 and O 2 atoms is not quite orthogonal to the $\mathrm{Cu}-\mathrm{O} 3$ vector that defines the pyramid vertex.

The apical $\mathrm{Cu}-\mathrm{O}$ distance is larger than those at the pyramid base due to the Jahn-Teller distortion normally associated with $\mathrm{Cu}^{2+}$ (Hewat, Fischer, Kaldis, Hewat, Jilek, Karpinski \& Rusiecki, 1990). On the other hand,
one would expect a Jahn-Teller distorted $\mathrm{Cu}^{2+}$ atom to occupy a larger volume than a $\mathrm{Cu}^{3+}$ atom that is not Jahn-Teller distorted. Formal charge balance requires Cu in $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ to be divalent. It is most unlikely to be in a mixed-valence state. From the data in Fig. 1 it is readily shown that the volume of the $\mathrm{CuO}_{5}$ pyramid in $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ is slightly smaller than that of the $\mathrm{Cu} 2 \mathrm{O}_{5}$ pyramid in $\mathrm{YBa}_{2} \mathrm{Cu}_{3} \mathrm{O}_{7-\delta}$. Vibrational data do not indicate that the latter group is anomalous, in any sense. The similarity of the Cu 2 atom in $\mathrm{YBa}_{2} \mathrm{Cu}_{3} \mathrm{O}_{7-\delta}$ to that indicated for the Cu atom in $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ by this accurate analysis argues against the hypothesis that the former atom is in a $3+$ state.

The $\mathrm{O} 1 \cdots \mathrm{O} 1, \mathrm{O} 1 \cdots \mathrm{O} 2$ and $\mathrm{O} 2 \cdots \mathrm{O} 2$ distances of 2.748 (2), 2.842 (1) and 2.779 (2) $\AA$, respectively, are comparable to the $\mathrm{O} \cdots \mathrm{O} 2.766 \AA$ distances in the $\mathrm{TiO}_{6}$ octahedra for $\mathrm{SrTiO}_{3}$ (Buttner \& Maslen, 1992). To first order, O 1 and O 2 behave as if they are in close non-bonded contact. The longer $\mathrm{O} 1 \ldots \mathrm{O} 3$ and $\mathrm{O} 2 \ldots \mathrm{O} 3$ distances of 3.285 (2) and 2.977 (1) $\AA$ raise the possibility that the cations play a significant role in defining the structural framework for $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$.

An approximate mirror line within the mirror plane at $y=0.75$ connects Ba atoms separated by the $c$ dimension, as shown in Fig. 2. The Ba atom is located on the pseudo-mirror line, the linear sequences $\mathrm{Y} 1 \cdots \mathrm{Cu} \cdots \mathrm{Y} 2 \cdots \mathrm{Ba}$ on both sides being related by the approximate mirror. That mirror is broken by O3. Off the exact mirror plane the approximate mirror persists as far as the closest O1, O2 pair. As seen from the atom coordinates projected onto the plane of Fig. 2, the deviation from symmetry is due to non-equivalence of O 3 on one side to an O 2 pair on the other.

The BaO bond strengths, indicated by their length, are affected by the number and strength of competing bonds to other cations. The Ba atom bonds tightly only to O 3 . There is a $\mathrm{Ba}-\mathrm{O} 3$ bond, 2.611 (1) $\AA$, on the $y=0.75$ mirror plane and two $m$-related bonds of 2.8312 (3) $\AA$. The O3 atom's apical bond to Cu is correspondingly weak, whereas Ol and O 2 form strong $\mathrm{Cu}-\mathrm{O}$ bonds.

The coordination geometries for both Y1 and Y2 correspond one to one, $\mathrm{Y}-\mathrm{O} 3$ being the shortest $\mathrm{Y}-\mathrm{O}$ bond in each case. Each $\mathrm{YO}_{7}$ coordination polyhedron


Fig. 1. Structural geometry of Cu pyramid (values in $\AA$ ): (a) $\mathrm{Y}_{2} \mathrm{BaCuO}_{5} ;$ (b) the superconducting phase $\mathrm{YBa}_{2} \mathrm{Cu}_{3} \mathrm{O}_{7-\gamma}$.
in $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ contains two mirror-related O 1 atoms and an O 3 atom, almost co-planar with the Y atom. The other two pairs of mirror-related O atoms complete the sevenfold Y -atom coordination.

In general, the contact distances indicate crowding of the Y cations in the mirror plane at $y=0.75$. Thus, the $\mathrm{Cu} \cdots \mathrm{Y} 2$ vector, 2.8230 (4) $\AA$, lies in the mirror plane, whereas the shortest $\mathrm{Cu} \cdots \mathrm{Y} 2$ vector between adjacent layers is 3.1464 (3) $\AA$. The Y1...Y2, 3.2846 (3) $\AA$, and Ba $\cdots \mathrm{Y} 1$ contacts, 3.9076 (3) $\AA$, are contracted by O3 involvement. The situation for the Ba atom is not clear cut. The shorter $\mathrm{Ba} \cdots \mathrm{Cu}$ contacts 3.2638 (4) and 3.3676 (4) $\AA$ lie in the $y=0.75$ plane, the former being contracted by interactions involving O3. On the other hand, the interplane $\mathrm{Ba} \cdots \mathrm{Y} 2,3.5321$ (3) $\AA$, vector is shorter than the in-plane $\mathrm{Ba} \cdots \mathrm{Y} 1,3.9076$ (3) $\AA$, vector, because Y2 is dragged towards Ba by the $\mathrm{Y} 2-\mathrm{Ol}$ and $\mathrm{Y} 2-\mathrm{O} 2$ interactions.

## 4. Atomic vibrations

Atomic vibrations are relevant here as indicators of the strengths of interactions. In general, the vibrational data are consistent with the view that the cations' motions are limited by strong $M-\mathrm{O}$ bonding, but all of the data are consistent with the hypothesis that such motions are also impeded appreciably by the stronger cation-cation interactions.


Fig. 2. $\Delta \rho$ in the (010) plane, $y=0.75$, of $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$. Map borders $12.2 \times 7.2 \AA$. Contour intervals 1.0 e $\AA^{-3}$; positive and negative contours are represented by solid lines and dashes, respectively.

Table 4. Atomic charges based on Hirshfeld partitioning of $\Delta \rho$

| Atom | Charge (e) | Alom | Charge (e) |
| :---: | :---: | :---: | :--- |
| Y1 | $2.3(1)$ | O1 | $-1.7(1)$ |
| Y2 | $2.0(1)$ | O2 | $-1.8(1)$ |
| Ba | $1.7(1)$ | O3 | $-1.7(1)$ |
| Cu | $1.8(1)$ |  |  |

The largest Cu -atom vibration is $\mathrm{ca} 45^{\circ}$ from both the $a$ and $c$ axes, creating angles of $15.9^{\circ}$ to the O 4 plane normal and $71.7^{\circ}$ to the $\mathrm{Y} 1 \cdots \mathrm{Cu} \cdots \mathrm{Y} 2$ line, suggesting that both the $\mathrm{Cu}-\mathrm{O}$ bonds and the $\mathrm{Cu} \cdots \mathrm{Y}$ contacts restrict the Cu -atom vibrations.

The O1-atom motion is closer to isotropic than those of O 2 and O 3 .

The coordination of the Cu and Y atoms around Ol is approximately tetrahedral, neglecting the weaker $\mathrm{Ba}-\mathrm{O} 1$ interactions. The O 1 atom's larger $U_{33}$ element can be attributed to a modest flattening of the tetrahedron in the $c$ direction. The coordination of the Cu and Y atoms around O 2 is closer to square planar. The O 2 atom has its largest motion along $\mathrm{Ba}-\mathrm{O} 2$ contacts normal to that plane. The coordination of O 3 with Cu , two Y atoms and the nearest Ba atom is also square planar. Its large $U_{22}$ vibration, being normal to the $y=0.75$ plane, indicates that O 3 vibrates maximally roughly along the two $m$-related $\mathrm{Ba}-\mathrm{O} 32.831 \AA$ vectors.

As O 1 and O 2 atoms are not pinned tightly at the corners of the O 4 plane, the $\mathrm{CuO}_{4}$ moiety oscillates about the normal plane. As $U_{i j}$ for O 1 are less than those for O 2 , the centre of rotation passes not through the central Cu atom itself, but nearer to O 1 than to O 2 .

The Ba atom is packed more loosely than Cu , as indicated by its larger $U_{\mathrm{eq}}$ value listed in Table 2. If the $\mathrm{Ba}-\mathrm{O} 3$ bond were very strong we should expect the minimum Ba -atom vibration along that bond, whereas the minimum vibration eigenvector makes an angle of $43.3^{\circ}$ with the $\mathrm{Ba}-\mathrm{O} 3$ vector. It is closer to the shortest $\mathrm{Ba} \cdots \mathrm{Cu}$ interaction and almost normal to the $\mathrm{Y} 2 \cdots \mathrm{Cu} \cdots \mathrm{Y} 1$ sequence, indicating that $\mathrm{Ba} \cdots$ cation interactions restrict the Ba -atom vibrations.

Tight packing results in the low vibrational parameters for both Y atoms listed in Table 2. The mean $\mathrm{Y}-\mathrm{O}$ bond length of $2.326 \AA$ for Y 2 , which has the lower $U_{\text {eq }}$ value, compares with $2.336 \AA$ for Y 1 . However, $\mathrm{Y}-\mathrm{O}$ bond lengths are not necessarily decisive, since Y2 is also involved in the shorter $M \cdots M^{\prime}$ contacts with both Ba and Cu . Both Y atoms vibrate maximally along the $b$ axis, but the anisotropy is less marked for Y2, which makes $\mathrm{Y} 2 \cdots \mathrm{Cu}$ contacts of 3.1464 (3) $\AA$ in the [010] direction. There is no similarly directed $\mathrm{Y} 1 \cdots$ cation vector.

The Y1 motion in the $x z$ plane is almost isotropic, as is consistent with having its three shortest $\mathrm{Y}-\mathrm{O}$ bonds almost at right angles. The Y2 atom vibrates least strongly along the $\mathrm{Y} 2-\mathrm{O} 3$ bond, which is almost coplanar with the two short Y2-O1 bonds. However, the degree of anisotropy of its motion in the $x z$ plane
is mild and its interactions with Cu and Y 1 in that plane may dampen its motion in the $c$ direction. The O atom vibration ellipsoids generally show hardness along the $\mathrm{Y}-\mathrm{O}$ bond directions and softness perpendicular to those bonds, except for $\mathrm{Y} 1-\mathrm{O}, 2.365$ (1), and $\mathrm{Y} 2-\mathrm{O}$, 2.309 (1) $\AA$, for which such softness may be precluded by other $\mathrm{Y}-\mathrm{O} 1$ or $\mathrm{Cu}-\mathrm{O}$ interactions.

## 5. Atomic charges

Atomic charges, determined by projecting the difference density $\Delta \rho$ onto atomic density basis functions following the method of Hirshfeld (1977), are listed in Table 4. There is close agreement between the charges for O1, O 2 and O 3 , and, to a lesser degree, between those for Y 1 and Y 2 . The O -atom charges are far more consistent than values determined earlier with Mo $K \alpha$ radiation (Buttner \& Maslen, 1993).

The O -atom values are reasonable for atoms formally in $2^{-}$states. The Cu -atom charge of +1.8 (1) e is likewise reasonable for a $\mathrm{Cu}^{\mathrm{II}}$ atom that overlaps strongly with its neighbours, as is indicated by its low $U_{i j}$ values. Crowding of atoms is associated with strong exchange depletion of the electron density and consequently with the development of a positive charge. The comparatively low positive charge of +1.7 e for the loosely bonded Ba atom is consistent with it having larger vibration amplitudes than the other cations. This reflects less transfer of valence electron density from cation to anion by exchange depletion. The higher charges on Y1 and Y2 are consistent with their lower vibration amplitudes, although it might also be argued that these reflect the higher oxidation states for the Y cations.

## 6. Difference density

Difference density ( $\Delta \rho$ ) maps based on the structure model in Table 2 with free-atom-scattering factors were evaluated. Electron density is transferred from the tightly packed regions of the structure, shown in Fig. 2, to form a lumpy sea in the space between the mirror planes at $y=0.25$ and 0.75 . The gradient of positive $\Delta \rho$ is low everywhere, i.e. the electron sea between the mirror planes is slowly varying. The major $\Delta \rho$ features do not have a simple relationship to the nearest-neighbour geometry. The complex $\Delta \rho$ topography suggests that the ligand field has significant contributions from both nearest- and second nearest-neighbour interactions.

Density is depleted most strongly along the shorter second nearest-neighbour cation-cation vectors. This is consistent with exchange depletion of valence electrons that overlap with closed electron sub-shells in the cations. There is very little positive $\Delta \rho$ on the $y=0.75$ section, or equivalently at $y=0.25$, on which all cations and the O 3 atom are packed tightly. The density at the

O3 site is close to zero. The featureless $\Delta \rho$ topography near this site contrasts with that nearer the cations.

Electron density is depleted especially along the sequence of short cation-cation contacts $\mathrm{Y} 1 \cdots \mathrm{Cu} \cdots \mathrm{Y} 2 \cdots \mathrm{Ba}$. There are weaker depletions near the Y1‥Y2 and the two symmetry-independent $\mathrm{Cu} \cdots \mathrm{Ba}$ vectors, forming cross-links between the $\mathrm{Y} 1 \cdots \mathrm{Cu} \cdots \mathrm{Y} 2 \cdots \mathrm{Ba}$ lines. Atoms in these lines are cross-linked by connections zigzagging along the $a$ axis. The Ba atom is on a pseudo-mirror line. Electron depletion between cations is broken at $\mathrm{Ba} \cdots \mathrm{Y} 1$, because of O 3 involvement, as shown in Fig. 2. The deviation from symmetry is due to the non-equivalence of O 3 , on one side, to an O 2 pair on the other.

Four second nearest-neighbour cations surround the Cu atom in the $y=0.75$ plane. Proceeding from the $\mathrm{Cu}-\mathrm{O} 3$ bond in a clockwise direction: $\mathrm{Cu} \cdots \mathrm{Ba} 3.2638$ (4), $\mathrm{Cu} \cdots \mathrm{Y} 1 \quad 2.9801$ (4), $\mathrm{Cu} \cdots \mathrm{Ba}$


Fig. 3. $\Delta \rho$ sections perpendicular to the $(010)$ plane for $\mathrm{Y}_{2} \mathrm{BaCuO}_{5}$ : (a) through $\mathrm{O}-\mathrm{O}$ contacts in $\mathrm{CuO}_{5}$; (b) through $\mathrm{Cu}-\mathrm{O}$ bonds. Map borders $7.9 \times 7.1 \AA$, contour intervals $0.5 \mathrm{e}^{\AA} \AA^{-3}$; positive and negative contours are represented by solid lines and dashes, respectively.
3.3676 (4) and $\mathrm{Cu} \cdots \mathrm{Y} 2$ 2.8230(4) $\AA$. Depletion is more pronounced along the $\mathrm{Cu} \cdots \mathrm{Y} 1, \mathrm{Cu} \cdots \mathrm{Y} 2$ and shorter $\mathrm{Cu} \cdots \mathrm{Ba}$ contacts. The two Ba and two Y atoms surrounding Cu are in approximately symmetric arrangements. Deviations from symmetry are due to the non-equivalence of O 3 , on one side, to the O 1 pair and O 2 pair on the other.

Four local minima surround the Cu nucleus; there is a $-2.9 \mathrm{e} \AA^{-3}$ minimum at the intersection of the $\mathrm{O} 1 \cdots \mathrm{O} 3 \cdots \mathrm{Ol}$ plane and the $\mathrm{Cu} \cdots \mathrm{Ba}$ vector, displaced $1.02 \AA$ from the Cu nucleus; $\mathrm{a}-3.7 \mathrm{e} \AA^{-3}$ hole, displaced $0.65 \AA$ along the $c$ axis, is tilted by $21^{\circ}$ from the $\mathrm{Cu} \cdots \mathrm{Y} 1$ vector; another $-1.3 \mathrm{e} \AA^{-3}$ minimum in the $\mathrm{O} 1 \cdots \mathrm{O} 2 \cdots \mathrm{O} 2 \cdots \mathrm{O} 1$ plane is displaced $0.6 \AA$ from the Cu nucleus; and the fourth, $-4.0 \mathrm{e} \AA^{-3}$ deep, $1.32 \AA$ from the Cu nucleus in the $\mathrm{O} 2 \cdots \mathrm{O} 3 \cdots \mathrm{O} 2$ plane, is tilted $6^{\circ}$ from the $\mathrm{Cu} \cdots \mathrm{Y} 2$ vector. Where such minima in the $y=$ 0.75 plane are not aligned accurately along cation-cation vectors, the topography can be interpreted in terms of movement of the depletion away from O3, that is of density towards that atom. To a minor extent, the O anion influences the deformation density.

Fig. 3(a), a $\Delta \rho$ section perpendicular to the (010) plane through the $\mathrm{O} 1 \cdots \mathrm{O} 2 \cdots \mathrm{O} 2 \cdots \mathrm{O} 1$ contacts, shows that electrons accumulate along the $\mathrm{O} 1 \cdots \mathrm{O} 2$ contacts. Less deformation density accumulates around O 1 , which from $U_{i j}$ moves less than O 2 .

The $\Delta \rho$ section perpendicular to the (010) plane passing through the $\mathrm{Cu}-\mathrm{O} 3$ bond in Fig. 3(b) shows that the slowly varying electron sea between the mirror planes accumulates along all the $\mathrm{O} \cdots \mathrm{O}$ contacts. The approximately symmetric $\Delta \rho$ topography around the Cu nucleus might have been expected from the characteristics of $s$ - and $d$-valence electron orbitals for this atom. An electron valley, of 3.5606 (3) $\AA$, links Cu at $y=0.75$ with Ba at $y=0.25$. The corresponding valley along the $\mathrm{Cu} \cdots \mathrm{Y} 2$ vector exists, but is less prominent.

The ligand field energy for the $3 d$ group of elements is larger than the spin-orbit coupling. The pure $3 d$ states are split by such a field into a lower triplet $t_{2 g}$ and upper doublet $e_{g}$ (Abraham \& Bleaney, 1970). Fig. 3(a) shows that the electron density does deplete near the Cu atom along the $\mathrm{Cu}-\mathrm{O} 1$ and $\mathrm{Cu}-\mathrm{O} 2$ vectors. Well defined depletions on either side of the Cu nucleus along the line of the $\mathrm{Cu}-\mathrm{O} 3$ vector also occur, as shown in Fig. $3(b)$. One could argue that these represent the $e_{g}$ orbital depletions expected for a $\mathrm{Cu}^{2+}$ atom, but as seen from Fig. 2 they relate more closely to the second nearestneighbour $\mathrm{Cu} \cdots M$ interactions than to the $\mathrm{Cu}-\mathrm{O}$ bonds. As the $e_{g}$ orbitals are depleted one might expect that density would concentrate in the corresponding $t_{2 g}$ states. This is harder to identify in the $\Delta \rho$ map, but mild $t_{2 g}$ concentration could be masked by other contributions.

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[^0]:    * A list of structure factors has been deposited with the IUCr (Reference: AS0699). Copies may be obtained through The Managing Editor, International Union of Crystallography, 5 Abbey Square, Chester CHI 2HU, England.

